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MODELLING OF GROUP EXPERT JUDGMENTS UNDER CONDITIONS OF COMPLEX UNCERTAINTY

Abstract: For decision making under uncertainty multicriteria decision methods are used to rank the alternatives based on the preferences expressed by experts. Decisions are often made on the basis of imprecise, uncertain and/or incomplete information provided by several sources, some of which may be unreliable. The paper offers implementation examples of the theory of plausible and paradoxical reasoning and the PCR5 rule for decisions making under conditions of complex uncertainties that are generated by arbitrary (contradictory) expert evidence. The mathematical models under consideration prevent errors and provide the greatest accuracy in processing the results of an expertise under conditions of multicriteria, multi-alternatives and complex uncertainties. The application of the classic combination rule and the PCR5 rule is used to deal with conflicting expert judgments of the Dezert-Smarandache model and the Dempster-Shafer model respectively.

Key words: DSmT, DST, frame of discernment, fusion, PCR5, uncertainty.

Problem statement. Effective results of group expert assessments analysis can be obtained with the suitable approach to pairwise comparison. The choosing of such an approach requires taking into account different forms of ignorance, i.e. situations where the necessary information is either missing or inadequate or presented in an inappropriate form. There are three common forms of ignorance: **incompleteness** – situations in which some data is unknown, but all available information is complete and correct; **inaccuracy** – a situation in which the reliability of data is certain, but the data itself is inaccurate; **uncertainty** – situations in which all available data is either true or false which can be estimated using probabilistic estimates.

In practice, different forms of ignorance, for instance, a combination of inaccuracy and uncertainty, occur rather often. Let’s consider a situation where for a certain expertise the methods for obtaining and/or analyzing expert information were chosen unreasonably. This factor generates the inaccuracy of the received data. Also in the process of analyzing expert judgments the information about the experts’ competence was not considered (or was not provided). This factor generates the uncertainty about the received data.

It is also necessary to take into account the fact that the obtained expert judgments (evidence) can interact with each other in different ways. Such pieces of evidence can be consistent ($\omega_1 \cap \omega_2 \cap \ldots \cap \omega_n \neq \varnothing$), consonant ($\omega_1 \subseteq \omega_2 \subseteq \ldots \subseteq \omega_n$), arbitrary ($\omega_1 \cap \omega_2 \cap \ldots \cap \omega_i \cap \ldots \cap \omega_n = \varnothing, \exists C: \omega_i \cap \omega_j \neq \varnothing$) or disjoint ($\omega_i \cap \omega_j = \varnothing$). Therefore, an application of new theories and approaches for the analysis of such expert judgments is needed [10, p.43; 11, p.37; 13, p.24].

Review. Various methods of group expert evaluation are widely used in the process of alternatives selection in conditions of weakly structured or unstructured problems. Consequently, the task of obtaining generalized expert assessments may be the basis for formation of the recommendations for the decision maker. In this case, the main problem is the processing of conflicts, which refers to situations where focal elements (selected subsets or groups of expert evidence) do not intersect.

The main cause of conflicts between basic belief assignments is the inconsistency of individual groups of expert evidence. Unfortunately, the fusion rules based on conjunction consensus do not take into account the degree of intersection of the initial focal elements [12, p.170].

In this paper three theories are considered, that are dedicated to the modelling of complex uncertainties, characterized by the union and intersection as forms of experts’ judgments interaction. They are as follows: Dempster-Shafer theory of evidence [1, p.150; 3, p.13; 7, p.3], Dezert-Smarandache theory of plausible and paradoxical reasoning [2, p.15; 5, p.11], conflict redistribution theory [6, p.24; 8, p.230].

The paper [8, p.207] provides information on how a number of combination methods handle conflicts. That can be summarized as follows:

– the Dempster fusion rule generally ignores the conflict and attributes any probability mass associated with conflict to the null set. The conflicting masses are used in the normalization process of the resulting masses;

– the Dezert-Smarandache classic fusion rule simply determines the combined probability masses for all possible sets of the initial focal elements, ignoring the nature of these intersections;

– in the DSm hybrid fusion rule, the transfer of partial conflicts (taking into account all integrity constraints of the model) is done directly onto the most specific sets including the partial conflicts but without proportional redistribution.

The rules of combination that are commonly used in the frameworks of DST and DSmT are as follows: the general Weighted Operator (WO), the Weighted Average Operator (WAO), minC rule, PCR1-PCR5 rules etc.

The basic idea of these rules is to redistribute total conflicting mass among the non-empty intersections of the initial focal elements. The main steps in applying all the rules of combination are as follows [6, p.2]:

– use the conjunctive rule to compute masses of non-empty sets of initial focal elements' intersections;

– compute the conflicting masses (partial and/or total);

– redistribute the conflicting masses to non-empty sets. This process differs for each of the PCR1-PCR5 rules.

The aim of the research is to consider the possibility of applying the theory of plausible and paradoxical reasoning and the theory of evidence for decision making under conditions of complex uncertainties that are generated by contradictory expert judgments.

Basic material and examples. Let $\Omega=\{\omega_i \mid i=\overline{1,n}\}$ be a finite discrete set of exhaustive and exclusive elements (hypotheses) called elementary elements in DST. Ω has been called the frame of discernment of hypotheses or universe of discourse. An expert may select subsets $X_j \subseteq \Omega$, $j=\overline{1,2^{|\Omega|}}$ including the empty set $\varnothing$, where 2^Ω is a cardinality of the power set. Any element of 2^Ω is a composite event (disjunction) of the frame of discernment that satisfies the following requirements:

- $X_j=\{\omega_i\}$ – an expert selected only one alternative $\omega_i \in \Omega$;
- $X_j=\{\omega_i \mid i=\overline{1,p}\},\ p<n$ – an expert selected p alternatives $\omega_i \in \Omega$; (1)
- $X_j=\Omega=\{\omega_i \mid i=\overline{1,n}\}$ – all alternatives are equivalent.

In real-life situations it is not always possible to correctly identify (distinguish) all or some elements of the frame of discernment (the condition of alternatives being mutually exclusive is not supported), in which case they can largely overlap one another. This is possible in a situation where the elements of the frame of discernment represent vague uncertain concepts, for instance, color palette, etc.

A new broader theory was needed to analyze such situations under conditions of uncertainty. Such a theory, created at the beginning of the 21st century, was called the theory of plausible and paradoxical reasoning (Dezert-Smarandache Theory – DSmT). In the DSmT framework, a frame $\Omega=\{\omega_i \mid i=\overline{1,n}\}$ consists of a finite set of exhaustive hypotheses, which can potentially intersect. In the frame an expert can select subsets $X_i \subseteq D^\Omega, i=\overline{1,\left|D^\Omega\right|}$, which satisfy the following requirements:

- The above mentioned requirements (1);
- If $(X_i, X_j) \subset D^\Omega$, then $(X_i \cap X_j) \in D^\Omega$ and $(X_i \cup X_j) \in D^\Omega$.

Therefore, the results of an expert evaluation are used to form a system of subsets $X=\{P_j \mid j=\overline{1,m}\}$, which displays all experts' judgments $E=\{E_j \mid j=\overline{1,m}\}$, where $P_j=\{X_i \mid i=\overline{1,p}\}$ – is a system of subsets, selected by a certain expert $E_j\,(p=\left|D^\Omega\right|-1, X_i \subseteq D^\Omega)$.

DSmT is based on so-called free DSm model. The exclusivity assumption between elements of Ω is not necessary within DSmT. However, in DSmT any integrity constraints between elements of Ω can also be introduced, if necessary, depending on the fusion problem under consideration. A free DSm model including some integrity constraints is called a hybrid DSm model. DSmT can deal also with the Shafer's model as well which appears actually only as a specific hybrid DSm model. The free model is formed on a set of all possible subsets of hyper-power set D^Ω.

For example, a frame of discernment $\Omega=\{\omega_1,\omega_2,\omega_3\}$ has $\left|D^\Omega\right|=19$ possible subsets. Therefore, subsets $X_i \subseteq \Omega, i=\overline{0,18}$ can

be formed based on one group of expert judgments. Experts assign *bbas* $m_i(X_i)$ to every X_i, which are combined using the DSm rule [5, p.16]:

$$m_{DS}(X) = \sum_{\substack{X_1,\dots,X_s \subset D^\Omega \\ X_1\cap\dots\cap X_s = X}} \prod_{i=1}^{s} m_i(X_i).$$

This rule is also known as conjunctive consensus between *bbas* for $s \geq 2$ independent sources of evidence.

The main disadvantage of the free model is that even with $n \leq 4$ elements of the frame of discernment, the calculations are very complicated. However, real-world tasks are characterized by much smaller number of alternatives than the number of all possible subsets D^Ω. Hence, the authors of the theory proposed a hybrid model, which is formed on the assumption of introducing restrictions on various elements of the free model.

There are three basic functions in DST and DSmT:

- the basic belief assignment (*bba*) $m : \Lambda \to [0;1]$:

$$0 \leq m(X_i) \leq 1, \quad \forall (X_i \in \Lambda), \quad m(\varnothing) = 0, \quad \sum_{X_i \in \Lambda} m(X_i) = 1;$$

- the belief function $Bel : \Lambda \to [0;1]$:

$$Bel(B) = \sum_{X_i \subseteq B, X_i \in \Lambda} m(X_i);$$

- the plausibility function $Pl : \Lambda \to [0;1]$:

$$Pl(B) = \sum_{X_i \cap B \neq \varnothing, X_i \in \Lambda} m(X_i),$$

where Λ is a power set which equals 2^Ω for DST and D^Ω for DSmT.

The value of $m(X_i)$ expresses the proportion of all relevant and available evidence that supports the claim that a particular element of Ω belongs to the set $X_i \subseteq \Omega$ but to no particular subset of X_i. The value of $m(X_i)$ pertains only to the set X_i and makes no additional claims about any subsets of X_i.

Bel(*B*) summarizes all our reasons to believe in *B* (i.e. the lower probability to believe in *B*).The plausibility *Pl*(*B*) of *B* measures the total belief mass that can move into *B* (interpreted sometimes as the upper probability of *B*). Since *Bel*(*B*) summarizes all our reasons to believe in *B* and *Pl*(*B*) expresses how much we should believe in *B* if all currently unknown were to support *B*, the true belief in *B* is somewhere in the following interval:

$$[Bel(B), Pl(B)] \subseteq [0,1].$$

Thus, the above mentioned models that illustrate the various forms of group expert judgments can be sources of specific uncertainties. Therefore, the traditional probability theory cannot be used for their modelling.

The PCR rules redistribute partial conflicting masses differently. The PCR combination rules work for any degree of conflict, for any DSm models (Shafer's model, free DSm model or any hybrid DSm model). The sophistication, complexity and also correctness of proportional conflict redistribution increases from the first PCR1 rule up to the last rule PCR5 [6, p.12].

Let's Ω be the frame of the fusion problem under consideration with two focal elements X_1 and X_2, and belief assignments: $m_1(X_1)$, $m_2(X_1)$, $m_1(X_2)$, $m_2(X_2)$. The focal elements are involved in a conflict, i.e. $X_1 \cap X_2 = \varnothing$. The conjunctive consensus is calculated as follows:

$$m(X_1 \cap X_2) = m_1(X_1) * m_2(X_2) + m_2(X_1) * m_1(X_2).$$

The PCR5 rule is based on the redistribution of the partial conflicting mass proportionally on non-empty sets involved in the conflict: $m(X_1 \cap X_2) : x = m_1(X_1) * m_2(X_2),\ \ y = m_2(X_1) * m_1(X_2).$ The conflicting mass *x* is redistributed to X_1 and X_2 proportionally with the masses $m_1(X_1)$ and $m_2(X_2)$ assigned to X_1 and X_2 respectively, the conflicting mass *y* is redistributed proportionally with the masses $m_2(X_1)$ and $m_1(X_2)$ assigned to X_1 and X_2 respectively.

For the fusion of *s*=2 sources the PCR5 is defined by [8, p.264]:

$$m_{PCR5}(X_1) = m(X_1) + \sum \left[\frac{m_1(X_1) * m_2(X_2)}{m_1(X_1) + m_2(X_2)} + \frac{m_2(X_1) * m_1(X_2)}{m_2(X_1) + m_1(X_2)} \right],$$

where $m(X_1)$ is a combined conflict mass of X_1, calculated on the basis of conjunctive consensus.

Example 1. To illustrate DSmT, let us consider a brief example. There is a frame of discernment $\Omega = \{\omega_1, \omega_2\}$ and a free DSm model:

$$X_0 = \varnothing, \quad X_1 = \omega_1, \quad X_2 = \omega_2, \quad X_3 = \omega_1 \cap \omega_2, \quad X_4 = \omega_1 \cup \omega_2$$

with associated *bbas*:

$$m_1(\omega_1) = 0{,}45; \quad m_1(\omega_2) = 0{,}3; \quad m_1(\omega_1 \cap \omega_2) = 0{,}1; \quad m_1(\omega_1 \cup \omega_2) = 0{,}15;$$

$$m_2(\omega_1) = 0{,}15; \quad m_2(\omega_2) = 0{,}35; \quad m_2(\omega_1 \cap \omega_2) = 0{,}4; \quad m_2(\omega_1 \cup \omega_2) = 0{,}1;$$

$$m_3(\omega_1) = 0{,}25; \quad m_3(\omega_2) = 0{,}2; \quad m_3(\omega_1 \cap \omega_2) = 0{,}2; \quad m_3(\omega_1 \cup \omega_2) = 0{,}35.$$

These masses are to be combined using the Dezert-Smarandache rule. All the combined subsets of the focal elements' intersections are presented in the table 1.

Table 1.

Non-empty sets of the three sources example

C_1C_2 \ C_3	ω_1	ω_2	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_1 \cap \omega_1$	ω_1	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	ω_1
$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_1 \cap \omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_1 \cap (\omega_1 \cup \omega_2)$	ω_1	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$	ω_1
$\omega_2 \cap \omega_1$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_2 \cap \omega_2$	$\omega_1 \cap \omega_2$	ω_2	$\omega_1 \cap \omega_2$	ω_2
$\omega_2 \cap \omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_2 \cap (\omega_1 \cup \omega_2)$	$\omega_1 \cup \omega_2$	ω_2	$\omega_1 \cup \omega_2$	ω_2
$\omega_1 \cap \omega_2 \cap \omega_1$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_1 \cap \omega_2 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_1 \cap \omega_2 \cap \omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$
$\omega_1 \cap \omega_2 \cap (\omega_1 \cup \omega_2)$	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$
$(\omega_1 \cup \omega_2) \cap \omega_1$	ω_1	$\omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$	ω_1
$(\omega_1 \cup \omega_2) \cap \omega_1$	$\omega_1 \cap \omega_2$	ω_2	$\omega_1 \cup \omega_2$	ω_2
$(\omega_1 \cup \omega_2) \cap \omega_1 \cap \omega_2$	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$
$(\omega_1 \cup \omega_2) \cap (\omega_1 \cup \omega_2)$	ω_1	ω_2	$\omega_1 \cup \omega_2$	$\omega_1 \cup \omega_2$

The calculated generalized *bbas* of expert evidence are as follows:

$$m_{DS}(\omega_1) = m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_1) + m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_1 \cup \omega_2) + m_1(\omega_1) *$$
$$* m_2(\omega_1 \cup \omega_2) * m_3(\omega_1) + m_1(\omega_1) * m_2(\omega_1 \cup \omega_2) * m_3(\omega_1 \cup \omega_2) + + m_1(\omega_1 \cup \omega_2) *$$
$$* m_2(\omega_1) * m_3(\omega_1) + m_1(\omega_1 \cup \omega_2) * m_2(\omega_1) * m_3(\omega_1 \cup \omega_2) + m_1(\omega_1 \cup \omega_2) *$$
$$* m_2(\omega_1 \cup \omega_2) * m_3(\omega_1) = 0{,}085;$$

$$m_{DS}(\omega_2) = m_1(\omega_2) * m_2(\omega_2) * m_3(\omega_2) + m_1(\omega_2) * m_2(\omega_2) * m_3(\omega_1 \cup \omega_2) +$$
$$+ m_1(\omega_2) * m_2(\omega_1 \cup \omega_2) * m_3(\omega_2) + m_1(\omega_2) * m_2(\omega_1 \cup \omega_2) * m_3(\omega_1 \cup \omega_2) +$$
$$+ m_1(\omega_1 \cup \omega_2) * m_2(\omega_2) * m_3(\omega_2) + m_1(\omega_1 \cup \omega_2) * m_2(\omega_2) * m_3(\omega_1 \cup \omega_2) +$$
$$+ m_1(\omega_1 \cup \omega_2) * m_2(\omega_1 \cup \omega_2) * m_3(\omega_2) = 0{,}106;$$

$$m_{DS}(\omega_1 \cap \omega_2) = 0{,}477;$$

$$m_{DS}(\omega_1 \cup \omega_2) = 0{,}332.$$

The calculated results indicate that the best alternative is intersection of the elements ω_1 and ω_2 with the *bba* that equals 0,477.

Example 2. For the next example let us consider the frame of discernment $\Omega = \{\omega_1, \omega_2, \omega_3\}$, Shafer's model, and the 3 following *bbas*:

$$m_1(\omega_1) = 0{,}15; \quad m_1(\omega_2) = 0{,}55; \quad m_1(\omega_3) = 0{,}3;$$

$$m_2(\omega_1) = 0{,}45; \quad m_2(\omega_2) = 0{,}35; \quad m_2(\omega_3) = 0{,}2;$$

$$m_3(\omega_1) = 0{,}25; \quad m_3(\omega_2) = 0{,}4; \quad m_3(\omega_3) = 0{,}35.$$

Fusion of the non-empty sets *bbas* of the initial focal elements yields:

$$m(\omega_1) = m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_1) = 0{,}15 * 0{,}45 * 0{,}25 = 0{,}017,$$

$$m(\omega_2) = m_1(\omega_2) * m_2(\omega_2) * m_3(\omega_2) = 0{,}55 * 0{,}35 * 0{,}4 = 0{,}077,$$

$$m(\omega_3) = m_1(\omega_3) * m_2(\omega_3) * m_3(\omega_3) = 0{,}3 * 0{,}2 * 0{,}35 = 0{,}021$$

with conflicting mass $k = 0{,}885$, therefore PCR5 rule is to be used.

The conflicting masses are:

$$m(\omega_1 \cap \omega_1 \cap \omega_2) = m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_2) = 0{,}027;$$

$$m(\omega_1 \cap \omega_1 \cap \omega_3) = m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_3) = 0{,}024;$$

$$m(\omega_1 \cap \omega_2 \cap \omega_1) = m_1(\omega_1) * m_2(\omega_2) * m_3(\omega_1) = 0{,}013;$$

$$m(\omega_1 \cap \omega_2 \cap \omega_2) = m_1(\omega_1) * m_2(\omega_2) * m_3(\omega_2) = 0{,}021;$$

$$m(\omega_1 \cap \omega_2 \cap \omega_3) = m_1(\omega_1) * m_2(\omega_2) * m_3(\omega_3) = 0{,}01838;$$

$$m(\omega_1 \cap \omega_3 \cap \omega_1) = m_1(\omega_1) * m_2(\omega_3) * m_3(\omega_1) = 0{,}0075;$$

$$m(\omega_1 \cap \omega_3 \cap \omega_2) = m_1(\omega_1) * m_2(\omega_3) * m_3(\omega_2) = 0{,}012;$$

$$m(\omega_1 \cap \omega_3 \cap \omega_3) = m_1(\omega_1) * m_2(\omega_3) * m_3(\omega_3) = 0{,}0105;$$

$$m(\omega_2 \cap \omega_1 \cap \omega_1) = m_1(\omega_2) * m_2(\omega_1) * m_3(\omega_1) = 0{,}062;$$

$$m(\omega_2 \cap \omega_1 \cap \omega_2) = m_1(\omega_2) * m_2(\omega_1) * m_3(\omega_2) = 0{,}099;$$

$$m(\omega_2 \cap \omega_1 \cap \omega_3) = m_1(\omega_2) * m_2(\omega_1) * m_3(\omega_3) = 0{,}0866;$$

$$m(\omega_2 \cap \omega_2 \cap \omega_1) = m_1(\omega_2) * m_2(\omega_2) * m_3(\omega_1) = 0{,}048;$$

$$m(\omega_2 \cap \omega_2 \cap \omega_3) = m_1(\omega_2) * m_2(\omega_2) * m_3(\omega_3) = 0{,}0674;$$

$$m(\omega_2 \cap \omega_3 \cap \omega_1) = m_1(\omega_2) * m_2(\omega_3) * m_3(\omega_1) = 0{,}0275;$$

$$m(\omega_2 \cap \omega_3 \cap \omega_2) = m_1(\omega_2) * m_2(\omega_3) * m_3(\omega_2) = 0{,}044;$$

$$m(\omega_2 \cap \omega_3 \cap \omega_3) = m_1(\omega_2) * m_2(\omega_3) * m_3(\omega_3) = 0{,}0385;$$

$$m(\omega_3 \cap \omega_1 \cap \omega_1) = m_1(\omega_3) * m_2(\omega_1) * m_3(\omega_1) = 0{,}03375;$$

$$m(\omega_3 \cap \omega_1 \cap \omega_2) = m_1(\omega_3) * m_2(\omega_1) * m_3(\omega_2) = 0{,}054;$$

$$m(\omega_3 \cap \omega_1 \cap \omega_3) = m_1(\omega_3) * m_2(\omega_1) * m_3(\omega_3) = 0{,}04725;$$

$$m(\omega_3 \cap \omega_2 \cap \omega_1) = m_1(\omega_3) * m_2(\omega_2) * m_3(\omega_1) = 0{,}02625;$$

$$m(\omega_3 \cap \omega_2 \cap \omega_2) = m_1(\omega_3) * m_2(\omega_2) * m_3(\omega_2) = 0{,}042;$$

$$m(\omega_3 \cap \omega_2 \cap \omega_3) = m_1(\omega_3) * m_2(\omega_2) * m_3(\omega_3) = 0{,}03675;$$

$$m(\omega_3 \cap \omega_3 \cap \omega_1) = m_1(\omega_3) * m_2(\omega_3) * m_3(\omega_1) = 0{,}015;$$

$$m(\omega_3 \cap \omega_3 \cap \omega_2) = m_1(\omega_3) * m_2(\omega_3) * m_3(\omega_2) = 0{,}024.$$

The first local conflict $m(\omega_1 \cap \omega_1 \cap \omega_2) = = m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_2)$ $= 0{,}15 * 0{,}45 * 0{,}4 = 0{,}027$ is proportionally redistributed between alternatives ω_1 and ω_2:

$$\frac{x_1}{0,15*0,45}=\frac{y_1}{0,4}=\frac{0,15*0,45*0,4}{0,15*0,45+0,4}.$$

Thus, $x_1 = 0,0039;\ y_1 = 0,0231.$

The second local conflict $m(\omega_1 \cap \omega_1 \cap \omega_3) == m_1(\omega_1) * m_2(\omega_1) * m_3(\omega_3)$ $= 0,15*0,45*0,35 = 0,024$ is proportionally redistributed between alternatives ω_1 and ω_3:

$$\frac{x_2}{0,15*0,45}=\frac{z_1}{0,35}=\frac{0,15*0,45*0,35}{0,15*0,45+0,35}.$$

Thus, $x_2 = 0,00382;\ z_1 = 0,01981.$

The local conflict $m(\omega_1 \cap \omega_2 \cap \omega_3) == m_1(\omega_1) * m_2(\omega_2) * m_3(\omega_3) =$ $0,15*0,35*0,35 = 0,0184$ is proportionally redistributed between alternatives ω_1, ω_2 and ω_3:

$$\frac{x_5}{0,15}=\frac{y_4}{0,35}=\frac{z_2}{0,35}=\frac{0,15*0,35*0,35}{0,15+0,35+0,35}.$$

Thus, $x_5 = 0,00324;\ y_4 = 0,00757;\ z_2 = 0,00757.$

Adding all corresponding masses, one gets the final result for ω_1, ω_2 and ω_3:

$$m_{PCR5}(\omega_1) = m(\omega_1) + x_1 + ... + x_{18} = 0,01688 + 0,0039 + 0,00382 + 0,00127 +$$
$$+ 0,01086 + 0,00324 + 0,00118 + 0,0024 + 0,00716 + 0,01051 + 0,06649 +$$
$$+ 0,02888 + 0,02719 + 0,00688 + 0,0092 + 0,02113 + 0,03831 + 0,00729 +$$
$$+ 0,0121 = 0,27868;$$

$$m_{PCR5}(\omega_3) = m(\omega_3) + z_1 + ... + z_{18} = 0,021 + 0,01981 + 0,00757 + 0,00632 +$$
$$+ 0,0032 + 0,00334 + 0,02246 + 0,04347 + 0,0055 + 0,02095 + 0,00435 +$$
$$+ 0,02455 + 0,01409 + 0,00894 + 0,00875 + 0,02864 + 0,00848 + 0,0029 +$$
$$+ 0,00313 = 0,25743.$$

$$m_{PCR5}(\omega_2) = m(\omega_2) + y_1 + ... + y_{18} = 0,077 + 0,0231 + 0,01185 + 0,01014 +$$
$$+ 0,00757 + 0,0064 + 0,05137 + 0,03251 + 0,03529 + 0,02094 + 0,02391 +$$
$$+ 0,01513 + 0,02305 + 0,03415 + 0,01878 + 0,01021 + 0,01336 + 0,02827 +$$
$$+ 0,02087 = 0,46389;$$

In this example, the initial level of experts' support regarding subset ω_3 on the frame of discernment $\Omega(m(\omega_1) = 0,017 \quad m(\omega_2) = 0,077 \ m(\omega_3) = 0,021)$ after the redistribution of conflict became even more pronounced.

Conclusion. This paper offers two examples of combination rules implementation. The differences between DSm and PCR5 rules lie in how they process the conflicting mass of empty sets. The classic DSm rule allocates these masses to the intersection of the corresponding focal elements. If the initial focal elements do not intersect, then the combined mass is allocated to both focal elements without any other assumptions. The general principle of the PCR5 rule is to redistribute the conflicting mass proportionally on non-empty sets involved in the model according to all integrity constraints. Thus, the PCR5 rule does the most accurate redistribution of partial conflict masses. It should also be noted that both the DSm and the PCR5 rules are hard to implement because they require a large amount of computation, especially in cases with more than two groups of evidence. However, the appropriate software makes such a disadvantage less significant. Perspective for further research is to consider presenting the task of searching and selecting multicriteria alternatives as the task of choosing the optimal organizational structures of complex organizational systems of large enterprises.

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